2017 Math Analysis Free Response Questions

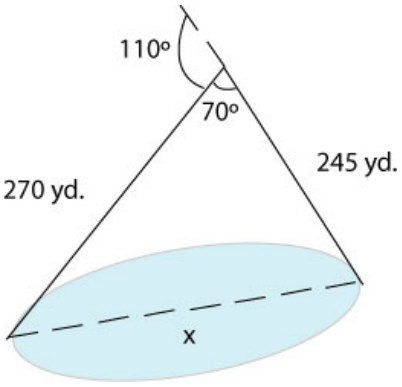
1. Brad invested $2,400 in an account that pays four percent compounded annually. How much will he have after 13 years?

2. A bacteria population doubles every eight minutes. If the population begins with one cell, how long will it take to grow to 512 cells?

3. Living organisms can be dated by the amount of carbon-14 present at time t compared to the amount present when the organism was alive. The half-life of carbon 14 is 5730 years. How long would it take for a 185 microgram sample of carbon-14 to decay to 24 grams? (Round to the nearest 100 years.)

4. Two scuba divers, Jim and Tara, are 20m apart below the surface of the water. They both spot a shark that is below them. The angle of depression from Jim to the shark is 47° and the angle of depression from Tara to the shark is 40°. How far is Jim from the shark?

5. To approximate the length of a lake, a surveyor starts at one end of the lake and walks 245 yards. He then turns 110º and walks 270 yards until he arrives at the other end of the lake. Approximately how long is the lake?



6. A 90-horsepower outboard motor at full throttle will rotate its propeller at 5000 rpm. Find the angular speed of the propeller in raidans per second.

7. The earth spins on its axis every 24 hours. If the earth's circumference is 24,800 miles, find the velocity of a person standing on the equator in miles per hour.

8.

RUNNING The table shows how wind affects a runner's performance in the 200 meter dash. Positive wind speeds correspond to tailwinds, and negative wind speeds correspond to headwinds. The change t in finishing time is the difference beween the runner's time when the wind speed is s and the runner's time when there is no wind.

Wind speed (m/sec), s	−6	−4	−2	0	2	4	6
Change in finishing time (sec), t	2.28	1.42	0.67	0	−0.57	−1.05	−1.42

a. Use a graphing calculator to find the best-fitting quadratic model.

b. Predict the change in finishing time when the wind speed is 10 m/sec.

Assignment

Date_____ Period____

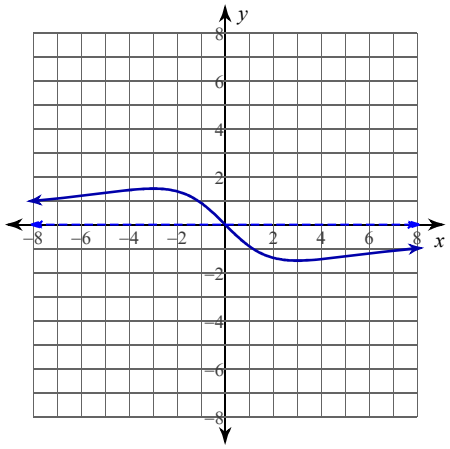
Find the intervals on which each function is continuous.

1) $f(x) = \dfrac{x}{x^2 + 10x + 25}$

A) $(-\infty, -4), (-4, -3), (-3, \infty)$
B) $(-\infty, -2), (-2, 0), (0, \infty)$
C) $(-\infty, \infty)$
D) $(-\infty, -5), (-5, \infty)$

Approximate the intervals where each function is increasing and decreasing.

2)



A) Increasing: $(0, 1.3)$ Decreasing: $(-\infty, 0), (1.3, \infty)$
B) Increasing: $(-0.7, 0), (0.7, \infty)$ Decreasing: $(-\infty, -0.7), (0, 0.7)$
C) Increasing: $(-\infty, 4)$ Decreasing: $(4, \infty)$
D) Increasing: $(-\infty, -3), (3, \infty)$ Decreasing: $(-3, 3)$

Use a graphing calculator to approximate the intervals where each function is increasing and decreasing.

3) $y = x^3 + 8x^2 + 21x + 18$

A) Increasing: $(-1, 0), (1, \infty)$ Decreasing: $(-\infty, -1), (0, 1)$
B) Increasing: $(-1, \infty)$ Decreasing: $(-\infty, -1)$
C) Increasing: $(-\infty, -3), (-2.3, \infty)$ Decreasing: $(-3, -2.3)$
D) Increasing: $(0, 1.3)$ Decreasing: $(-\infty, 0), (1.3, \infty)$

For each problem, find the average rate of change (slope) of the function over the given interval.

4) $f(x) = 2x^2 + 2x - 1; \quad [1, \dfrac{5}{4}]$

Transform the given function $f(x)$ as described and write the resulting function as an equation.

5) $f(x) = x^3$

expand horizontally by a factor of 3
reflect across the x-axis
translate left 3 units
translate up 2 units

A) $g(x) = -\left(\frac{1}{3}(x+3)\right)^3 + 2$

B) $g(x) = 3 \cdot (-(x+3))^3 + 2$

C) $g(x) = -\left(\frac{1}{3}(x+2)\right)^3 + 3$

D) $g(x) = -\frac{1}{3}(x-2)^3 - 2$

6) $f(x) = \sqrt{x}$

reflect across the y-axis
expand vertically by a factor of 2
reflect across the x-axis
translate right 2 units
translate down 3 units

A) $g(x) = \frac{1}{2}\sqrt{x-2} + 3$

B) $g(x) = \sqrt{2(x+3)} - 2$

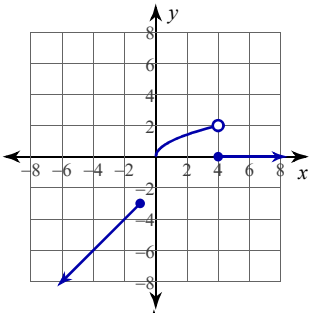
C) $g(x) = -2\sqrt{-(x-2)} - 3$

D) $g(x) = \sqrt{2(x-3)} - 2$

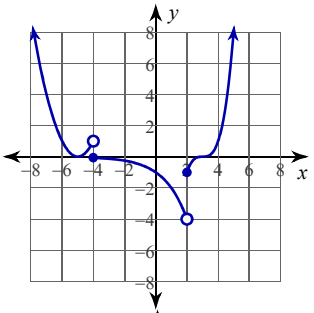
Sketch the graph of each function.

7) $f(x) = \begin{cases} x - 2, & x \le -1 \\ \sqrt{x}, & -1 < x < 4 \\ 0, & x \ge 4 \end{cases}$

A)



B)



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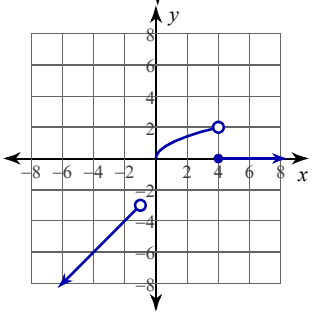
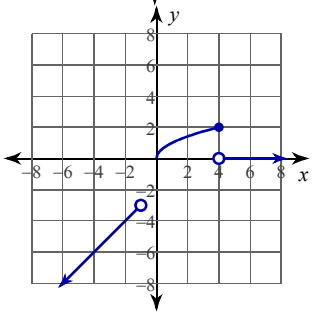


D)



Perform the indicated operation.

8) $g(t) = 3t - 4$
$h(t) = -t^3 + 2t^2$
Find $g(2) \cdot h(2)$

A) 0 B) −160 C) 45 D) 104

9) $g(x) = x^3 - 4x$
$f(x) = x + 1$
Find $(3g - 2f)(-4)$

A) −138 B) −81 C) 87 D) −6

Find the inverse of each function.

10) $f(x) = \frac{3}{x} + 1$

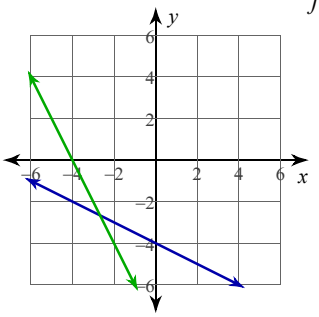
A) $f^{-1}(x) = \frac{3}{x - 1}$ B) $f^{-1}(x) = \frac{4}{-x + 1} - 2$

C) $f^{-1}(x) = \frac{1}{x + 1}$ D) $f^{-1}(x) = -\frac{2}{x + 2}$

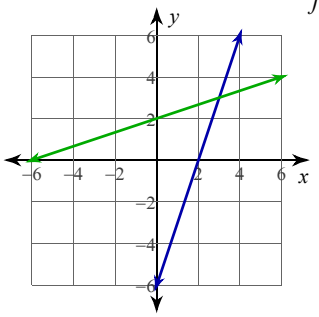
Find the inverse of each function. Then graph the function and its inverse.

11) $f(x) = -6x - 1$

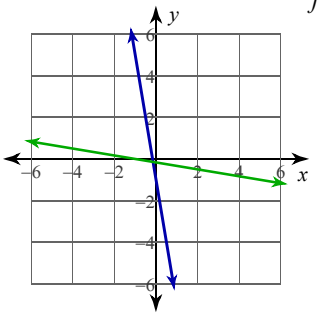
A) $f^{-1}(x) = -2x - 8$



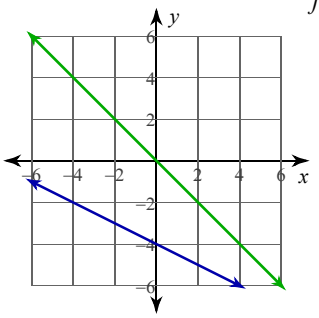
B) $f^{-1}(x) = 2 + \frac{1}{3}x$



C) $f^{-1}(x) = -\frac{1}{6}x - \frac{1}{6}$

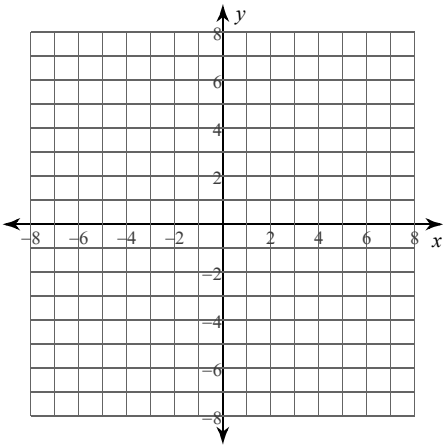


D) $f^{-1}(x) = -x$



For each function: (1) state the maximum number of turns the graph could make, (2) determine the real zeros and state the multiplicity of any repeated zeros, (3) list the x-intercepts where the graph crosses the x-axis and those where it does not cross the x-axis, (4) describe the end behavior, and (5) sketch the graph.

12) $f(x) = -2x^3 - 5x^2$



Use long division or synthetic division to determine if $d(x)$ is a factor of $f(x)$

13) $f(x) = x^2 + 4x + 4$
$d(x) = x + 2$

Find the remainder when $f(x)$ is divided by $x - k$.

14) $f(x) = 2x^3 - x^2 - 25x - 12$
$k = -1$

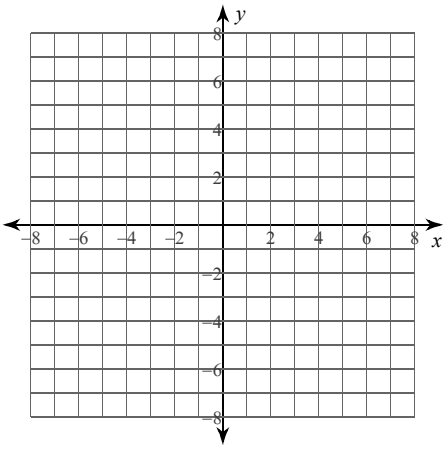
A) 1 B) −6 C) 10 D) −10

Write a polynomial function of least degree with integral coefficients that has the given zeros.

15) −5, 2, 4

For each function, identify the holes, intercepts, vertical and horizontal asymptote. Then sketch the graph.

16) $f(x) = \dfrac{x^2 + x - 2}{x^2 - 9}$



Solve each equation.

17) $4^{b-1} = \dfrac{1}{64}$

A) $\{5\}$ B) $\left\{-\dfrac{8}{5}\right\}$ C) $\{-2\}$ D) { All real numbers. }

Rewrite each equation in logarithmic form.

18) $m^n = 83$

A) $\log_m 83 = n$ B) $\log_n m = 83$ C) $\log_{83} n = m$ D) $\log_m n = 83$

19) $x^{-1} = 65$

A) $\log_{65} -1 = x$ B) $\log_{65} x = -1$ C) $\log_{-1} 65 = x$ D) $\log_x 65 = -1$

Use a calculator to approximate each to the nearest thousandth.

20) $\log_5 19$

A) 1.829 B) 1.999 C) 1.248 D) 1.074

Evaluate each expression.

21) $\log_3 81$

A) 4 B) 27 C) 3 D) 5

Condense each expression to a single logarithm.

22) $\log_3 y + \log_3 z + \log_3 w + \dfrac{\log_3 x}{2}$

23) Heather invests $4,224 in a savings account with a fixed annual interest rate of 8% compounded continuously. What will the account balance be after 4 years?

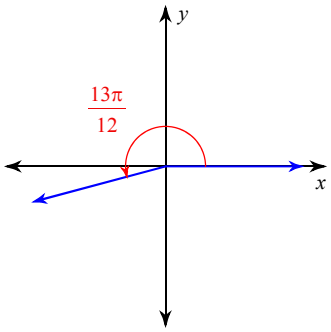
A) $4,907.59 B) $5,289.81 C) $5,701.80 D) $5,816.99

Find the reference angle.

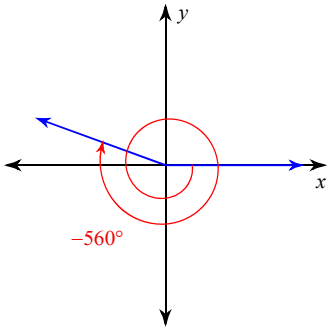
24) 475°

A) 70° B) 65° C) 10° D) 30°

25)



26)



Convert each degree measure into radians and each radian measure into degrees.

27) 345°

A) $\frac{23\pi}{12}$ B) $\frac{13\pi}{6}$ C) $\frac{65\pi}{36}$ D) $\frac{31\pi}{18}$

28) $-\frac{\pi}{6}$

A) -30° B) -40° C) -20° D) -35°

Find the value of the trig function indicated.

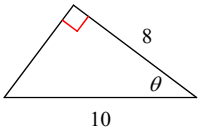
29) Find $\cot\theta$ if $\sin\theta = \frac{8}{17}$

A) $\frac{15}{8}$ B) $\frac{15}{17}$ C) $\frac{8}{15}$ D) $\frac{17}{15}$

30) Find $\sin\theta$ if $\cos\theta = \frac{3}{5}$

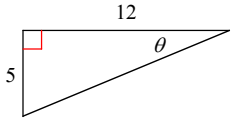
A) $\frac{24}{25}$ B) $\frac{4}{5}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$

31) $\sec\theta$



A) $\frac{5}{3}$ B) $\frac{3}{4}$ C) $\frac{5}{4}$ D) $\frac{4}{3}$

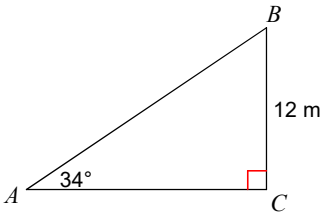
32) $\sin\theta$



A) $\frac{5}{13}$ B) $\frac{5}{12}$ C) $\frac{13}{12}$ D) $\frac{12}{5}$

Solve each triangle. Round answers to the nearest tenth.

33)



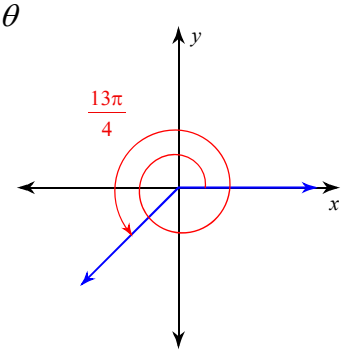
A) $m\angle B = 56.6°,\ b = 17.8$ m, $c = 21.5$ m
B) $m\angle B = 58.1°,\ b = 17.8$ m, $c = 21.5$ m
C) $m\angle B = 56°,\ b = 17.8$ m, $c = 21.5$ m
D) $m\angle B = 58.9°,\ b = 17.8$ m, $c = 21.5$ m

In each problem, angle C is a right angle. Find the side indicated to the nearest tenth.

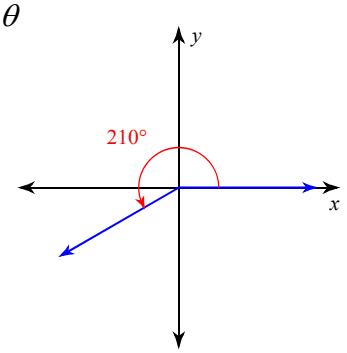
34) Find b if $c = 9,\ m\angle A = 26°$

Find the exact value of each trigonometric function.

35) sec θ

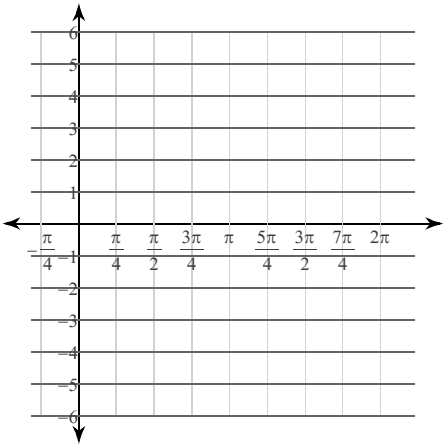


36) cot θ



Find the amplitude, the period in radians, the phase shift in radians, the vertical shift, the minimum and maximum values, and the transformations required to obtain the graph starting with a basic trig function. Then sketch the graph using radians.

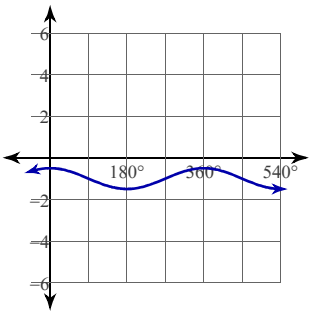
37) $y = 3\sin\left(2\theta + \frac{5\pi}{3}\right) + 2$



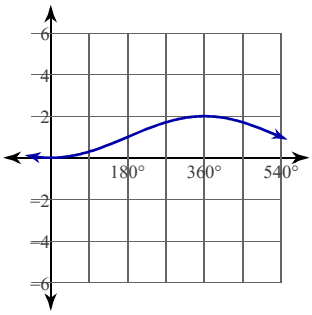
Graph each function using degrees.

38) $y = \frac{1}{2} \cdot \sin(\theta - 90) + 1$

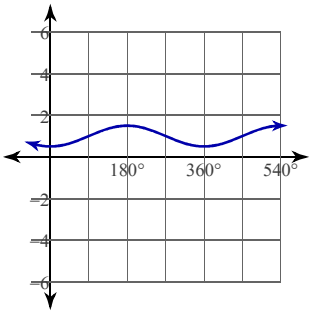
A)



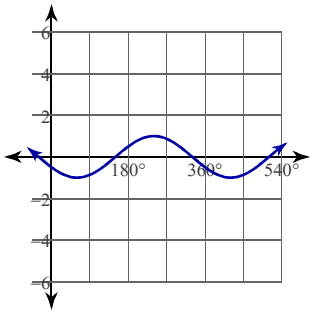
B)



C)



D)



Solve each equation for $0 \le \theta < 360$. Round your answers to the nearest hundredth.

39) $\frac{1}{2} = \sin\theta$

A) $\{0, 30, 180\}$ B) $\{30, 150\}$ C) $\{180\}$ D) $\{150\}$

Find the exact value of each expression.

40) $\sin^{-1}\frac{\sqrt{3}}{2}$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) 0 D) $\frac{\pi}{6}$

Verify each identity.

41) $\dfrac{1 + \csc^2 x}{\csc^2 x} = \sin^2 x + 1$

42) $\sec^2 x + \cot^2 x = \tan^2 x + \csc^2 x$

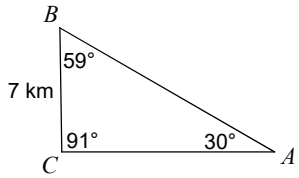
Simplify.

43) $\cos 3x \cos -6x + \sin 3x \sin -6x$

A) $\cos 9x$ B) $\sin -3x$ C) $\cos -3x$ D) $\cos -10x$

Solve each triangle. Round your answers to the nearest tenth.

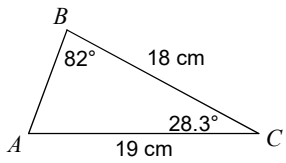
44)



A) $c = 14$ km, $b = 12$ km B) $c = 15$ km, $b = 12$ km

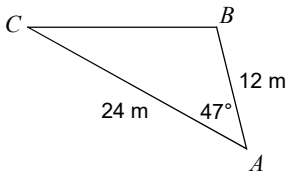
C) $c = 11$ km, $b = 13$ km D) $c = 14$ km, $b = 11$ km

45)



A) $m\angle A = 69.7°$, $c = 10$ cm
B) $m\angle A = 69.7°$, $c = 8$ cm
C) $m\angle A = 69.7°$, $c = 9.1$ cm
D) $m\angle A = 69.7°$, $c = 12$ cm

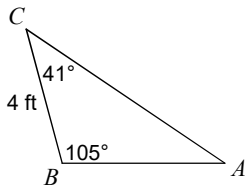
46)



A) $m\angle B = 104°$, $m\angle C = 29°$, $a = 18.1$ m
B) $m\angle B = 107.7°$, $m\angle C = 25.3°$, $a = 18.1$ m
C) $m\angle B = 121.5°$, $m\angle C = 11.5°$, $a = 21.2$ m
D) $m\angle B = 111.4°$, $m\angle C = 21.6°$, $a = 18.1$ m

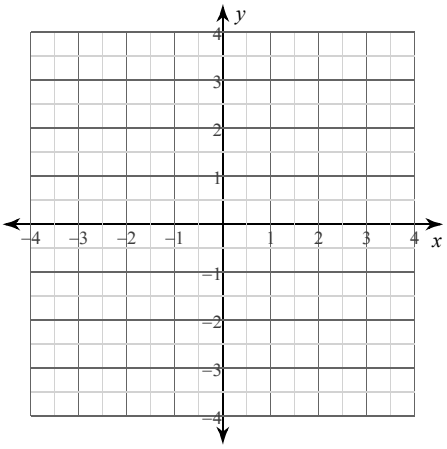
Find the area of each triangle to the nearest tenth.

47)



Convert each pair of polar coordinates to rectangular coordinates.

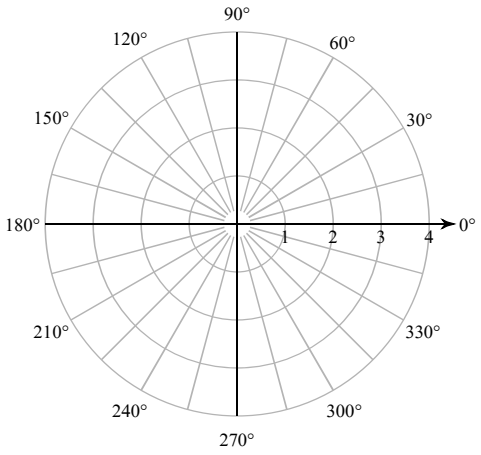
48) $\left(2, \frac{7\pi}{6}\right)$



A) $\left(-\sqrt{3}, -1\right)$ B) $\left(-1, \sqrt{3}\right)$ C) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ D) $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

Plot the point with the given polar coordinates.

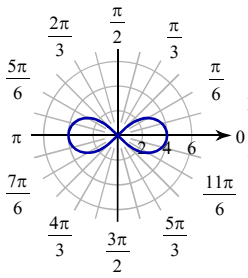
49) $(2, 255°)$



Consider each polar equation over the given interval. Classify the curve; determine if the graph is symmetric with respect to the origin, polar axis, and line $\theta=\pi/2$; find the values of θ where r is zero; find the maximum $|r|$ value and the values of θ where this occurs; and sketch the graph.

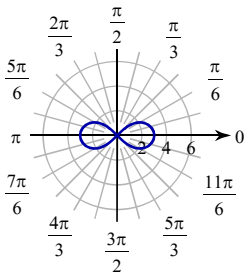
50) $r^2 = 36\cos(2\theta),\ 0 \le \theta < 2\pi$

A)



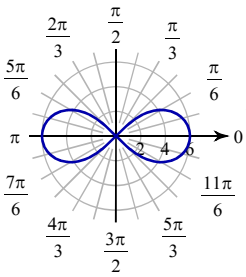
Lemniscate
Symmetric about the origin, polar axis, and line $\theta = \frac{\pi}{2}$
$r = 0$ when $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
$|r| = 4$ when $\theta = \{0, \pi\}$

B)



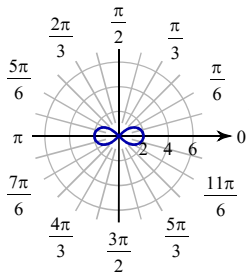
Lemniscate
Symmetric about the origin, polar axis, and line $\theta = \frac{\pi}{2}$
$r = 0$ when $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
$|r| = 3$ when $\theta = \{0, \pi\}$

C)



Lemniscate
Symmetric about the origin, polar axis, and line $\theta = \frac{\pi}{2}$
$r = 0$ when $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
$|r| = 6$ when $\theta = \{0, \pi\}$

D)



Lemniscate
Symmetric about the origin, polar axis, and line $\theta = \frac{\pi}{2}$
$r = 0$ when $\theta = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
$|r| = 2$ when $\theta = \{0, \pi\}$

Convert each equation from rectangular to polar form.

51) $(x+3)^2 + y^2 = 9$

A) $r = -6\sin\theta$ B) $r = -6\cos\theta$ C) $r = -4\cos\theta$ D) $r = 6\sin\theta$

Find the component form of the resultant vector.

52) $\mathbf{f} = \langle 1, -2 \rangle$
Find: $9\mathbf{f}$

A) $\langle 9, -18 \rangle$ B) $\langle 24, 60 \rangle$ C) $\langle 36, -48 \rangle$ D) $\langle -150, -360 \rangle$

53) $\mathbf{u} = \langle -5, -1 \rangle$
$\mathbf{g} = \langle -6, 9 \rangle$
Find: $-4\mathbf{u} + 4\mathbf{g}$

A) $\langle 36, 18 \rangle$ B) $\langle -4, 40 \rangle$ C) $\langle 72, -54 \rangle$ D) $\langle -155, 49 \rangle$

54) $\mathbf{u} = \langle -3, 5 \rangle$
Find: $10\mathbf{u}$

A) $\langle -24\sqrt{2}, -32\sqrt{2} \rangle$ B) $\langle 10\sqrt{2}, -3\sqrt{2} \rangle$ C) $\langle 12, -12\sqrt{3} \rangle$ D) $\langle -30, 50 \rangle$

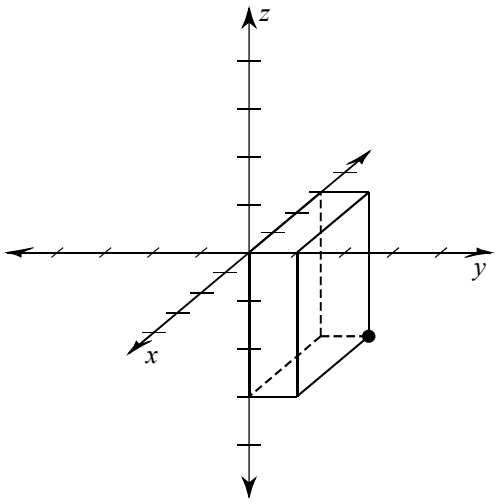
Find the dot product of the given vectors.

55) $\mathbf{u} = \langle 6, -8 \rangle$
$\mathbf{v} = \langle 9, 9 \rangle$

A) -40 B) -78 C) -71 D) -18

Write the coordinates of each point.

56)



A) $(3, 3, -1)$ B) $(1, 3, 3)$ C) $(3, 1, -3)$ D) $(-3, 1, -3)$

Write each vector in component form.

57) $\overrightarrow{RS}$ where $R = (-6, 9, -8)$ $S = (-5, 3, -3)$

Find the cross product of the given vectors.

58) $\mathbf{a} \times \mathbf{b}$
$\mathbf{a} = \langle 4, 4, 3 \rangle$
$\mathbf{b} = \langle 2, 4, -6 \rangle$

A) $\langle -36, 30, -8 \rangle$ B) $\langle -18, 8, -12 \rangle$ C) $\langle -24, 18, -36 \rangle$ D) $\langle -36, 30, 8 \rangle$

Simplify. Write "undefined" for expressions that are undefined.

59) $\begin{bmatrix} -6 & -2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 & -6 & -5 & -6 \\ -6 & 6 & -2 & 4 \end{bmatrix}$

A) $\begin{bmatrix} 36 & 24 & 34 & 44 \\ 2 & -12 & -3 & -2 \end{bmatrix}$

B) Undefined

C) $\begin{bmatrix} 36 & 24 & 34 & 28 \\ 2 & -12 & -3 & -10 \end{bmatrix}$

D) $\begin{bmatrix} 36 & 24 & 34 & 28 \\ -10 & 0 & -7 & -2 \end{bmatrix}$

60) $\begin{bmatrix} 6 & 3 & -2 \\ 2 & 6 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -6 \\ -3 & 1 & 1 \end{bmatrix}$

A) $\begin{bmatrix} 6 & 6 & -8 \\ -1 & 7 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 6 & 5 & -8 \\ -1 & 7 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 6 & 5 & -8 \\ -1 & 5 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 6 & 5 & 4 \\ -5 & 7 & 0 \end{bmatrix}$

Evaluate each determinant.

61) $\begin{vmatrix} -2 & 3 \\ -4 & -2 \end{vmatrix}$

Find the next three terms in each sequence.

62) −34, −27, −20, −13, −6, ...

A) −1, 6, 13 B) 1, 8, 15 C) 3, 11, 19 D) 4, 12, 20

Determine if the sequence is arithmetic. If it is, find the common difference, the 52nd term, and the explicit formula.

63) −12, −32, −52, −72, ...

A) Common Difference: $d = -22$
$a_{52} = -1155$
Explicit: $a_n = -11 - 22n$

B) Common Difference: $d = -20$
$a_{52} = -1032$
Explicit: $a_n = 8 - 20n$

C) Common Difference: $d = -21$
$a_{52} = -1104$
Explicit: $a_n = -12 - 21n$

D) Common Difference: $d = -21$
$a_{52} = -1083$
Explicit: $a_n = 9 - 21n$

Determine if the sequence is geometric. If it is, find the common ratio, the 8th term, and the explicit formula.

64) $-1,\ -5,\ -25,\ -125,\ ...$

A) Common Ratio: $r = -4$
$a_8 = 16384$
Explicit: $a_n = -(-4)^{n-1}$

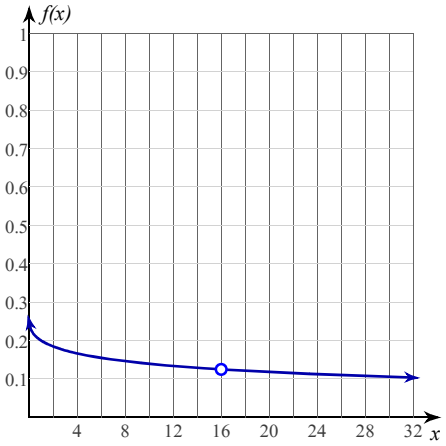
B) Common Ratio: $r = \frac{1}{4}$
$a_8 = \frac{1}{16384}$
Explicit: $a_n = \left(\frac{1}{4}\right)^{n-1}$

C) Common Ratio: $r = 4$
$a_8 = 16384$
Explicit: $a_n = 4^{n-1}$

D) Common Ratio: $r = 5$
$a_8 = -78125$
Explicit: $a_n = -5^{n-1}$

Evaluate each limit.

65) $\lim\limits_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$



66) $\lim\limits_{x \to 5^+} \frac{x^2}{5x - 25}$

A) ∞ B) -3 C) $-\infty$ D) -6

Use the definition of the derivative to find the derivative of each function with respect to *x*. (Hint: Another word for derivative is slope)

67) $y = 3x + 5$

A) $\frac{dy}{dx} = -4$ B) $\frac{dy}{dx} = 3x - 3$ C) $\frac{dy}{dx} = 3x + 3$ D) $\frac{dy}{dx} = 3$

Differentiate each function with respect to *x*. Use the Power Rule (Multiply coefficient by the exponent and subtract one from the exponent)

68) $y = 8x^8 + 4x^2$

A) $\frac{dy}{dx} = 64x^7 + 8x$ B) $\frac{dy}{dx} = 64x^8 + 8x^2$ C) $\frac{dy}{dx} = 64x + 8x$ D) $\frac{dy}{dx} = 8x^7 + 4x$

Expand completely.

69) $(b + 3a)^4$

A) $b^4 + 12b^3a + 54b^2a^2 + 108ba^3 + 81a^4$ B) $6b^4 + 45b^3a + 180b^2a^2 + 405ba^3 + 486a^4$

C) $b^4 + 3b^3a + 9b^2a^2 + 27ba^3 + 81a^4$ D) $b^4 + 12b^3a + 54b^2a^2 + 108ba^3 + 243a^4$

70) $(y - 3x)^3$

A) $y^3 - 9y^2x + 18yx^2$ B) $y^3 - 9y^2x + 27yx^2$

C) $y^3 - 9y^2x + 27yx^2 - 27x^3$ D) $y^3 - 3y^2x + 9yx^2 - 27x^3$

Answers to Assignment (ID: 1)

1) D

2) D

3) C

4) $\frac{13}{2}$

5) A

6) C

7) A

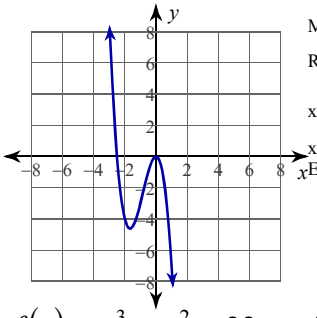
8) A

9) A

10) A

11) C

12)



Max # turns: 2

Real zeros: $\left\{0 \text{ mult. } 2, -\frac{5}{2}\right\}$

x-int, crosses: $-\frac{5}{2}$

x-int, doesn't cross: 0

End behavior:

$\lim_{x \to -\infty} f(x) = \infty$

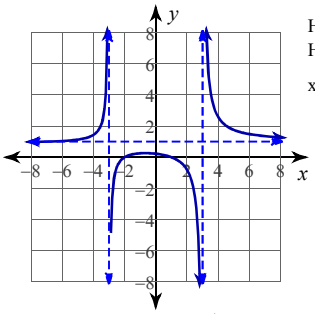
$\lim_{x \to \infty} f(x) = -\infty$

13) Yes

14) C

15) $f(x) = x^3 - x^2 - 22x + 40$

16)



Holes: None

Horz. Asym.: $y = 1$

x-intercepts: 1, −2, y-intercept: $\frac{2}{9}$

17) C

18) A

19) D

20) A

21) A

22) $\log_3 \left(wzy\sqrt{x}\right)$

23) D

24) B

25) $\frac{\pi}{12}$

26) 20°

27) A

28) A

29) A

30) B

31) C

32) A

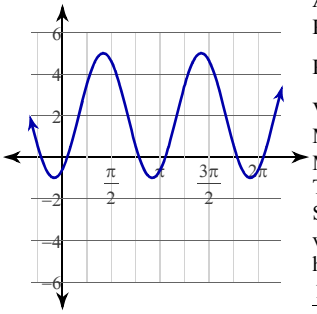
33) C

34) 8.1

35) $-\sqrt{2}$

36) $\sqrt{3}$

37)



Amplitude: 3

Period: π

Phase shift: Left $\frac{5\pi}{6}$

Vert. shift: Up 2

Min: −1

Max: 5

Transformations: Starting with sin θ, vertically stretch by 3, horizontally shrink by $\frac{1}{2}$, translate left $\frac{5\pi}{6}$, translate up 2

38) C

39) B

40) A

41) $\frac{1 + \csc^2 x}{\csc^2 x}$ Decompose into sine and cosine

$\frac{1 + \left(\frac{1}{\sin x}\right)^2}{\left(\frac{1}{\sin x}\right)^2}$ Simplify

$\sin^2 x + 1$ ■

42) $\sec^2 x + \cot^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$\tan^2 x + 1 + \cot^2 x$ Use $\cot^2 x + 1 = \csc^2 x$

$\tan^2 x + \csc^2 x$ ■

43) A

44) A

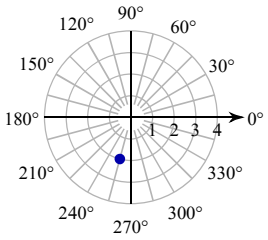
45) C

46) A

47) 9.1 ft²

48) A

49)



50) C

51) B

52) A

53) B

54) D

55) D

56) D

57) $\langle 1, -6, 5 \rangle$

58) D

59) C

60) B

61) 16

62) B

63) B

64) D

65) $\frac{1}{8}$

66) A

67) D

68) A

69) A

70) C